Mechanical Vibrations Equation Sheet

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# Free Vibrations without damping

* + General form:
    - General solution (3 forms):

# Viscous damping

* + - * Auxiliary equation solution

### Critical damping

* + - * When a system is critically damped
      * Critical damping

### Damping ratio

* + - * Recall
    - Solution to auxiliary equation

## Underdamped system

* + - therefore is negative and gives an imaginary number
    - Auxiliary equation solution
    - Damped natural frequency
    - General solution (3 forms):

## Overdamped system

* + - so the determinant is positive
    - Auxiliary equation system
    - General solution

## Critically damped system

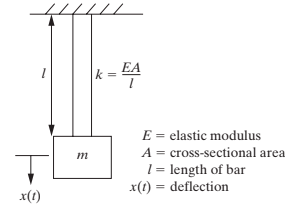
* + - so determinant is zero
    - Auxiliary equation system
    - General solution

## Experimental identification

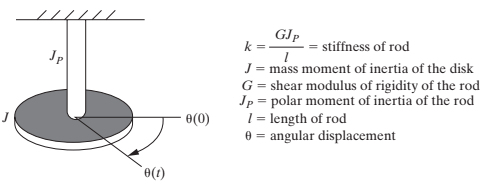
* + - * + Where is one peak and
        + is the directly next peak
    - * is the time between peaks

# Estimating stiffness of mechanical elements

## Vertical beam

* + - 

## Shaft

* + - 

## Cantilever beam

## Simply supported beam

* + - If

## Spring

* + - * diameter of coil material
      * number of turns
      * = diameter of turn

# Springs

* + Series
  + Parallel

# Modeling using energy method

* + - Spring (potential energy)
    - Mass (kinetic energy)
    - Loss of energy (damper)

# Stability

* + Oscillating system instability

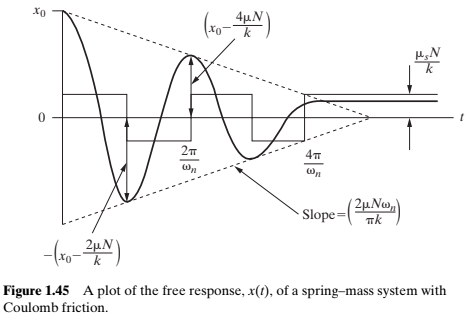
## Flutter instability

* + Non-oscillating system direct instability

## Divergent instability

* + - If
      * We have our traditional stable system
    - If
      * + Flutter instability
        + Divergent instability

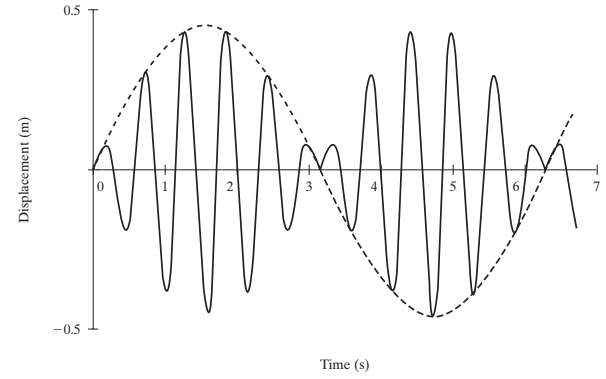
# Coulomb friction

* + - 

# Numerical simulations

* + Say and
  + Then becomes
  + We can now have a linear system of equations:
    - * We can solve this numerically

# Harmonic excitation without damping

* + - Note that
  + Method of undetermined coefficients leads to
  + Resonance
    - If
    - Rearrange equation and using some L’Hopital
  + Beating
    - If
    - Rearrange using and trig rule
    - and
      * 

# Harmonic excitation with damping

* + - Rearrange into
  + Transient response we multiplied by the and dies out over time

## Steady state response

* + - * The non-exponentially decaying terms
  + The particular solution can also be written as
    - We can study this equation to look at what happens at
      * Resonance can’t happen though because the equation is fully defined for non-zero frequencies
        + Say

## Magnification factor

* + - * + Recall the deformation
        + Graph

